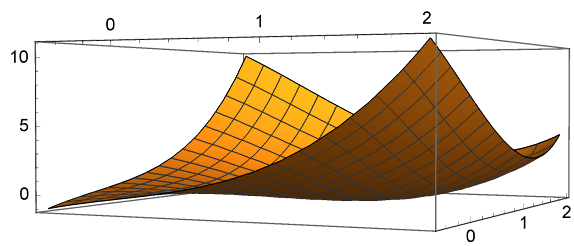
```
(* We find and characterize the critical points of a function f:R^2→R *)
```

In[196]:=
```
f = x^3 - 3 x y + y^3
xmin = -.5 ;
xmax = 2 ;
ymin = -.5 ;
ymax = 2;
Gf = Grad[f, {x, y}] (* gradient of f *)
Plot3D[f, {x, xmin, xmax}, {y, ymin, ymax}]
ContourPlot[f, {x, xmin, xmax}, {y, ymin, ymax}]
VectorPlot[Gf, {x, xmin, xmax}, {y, ymin, ymax}]
```

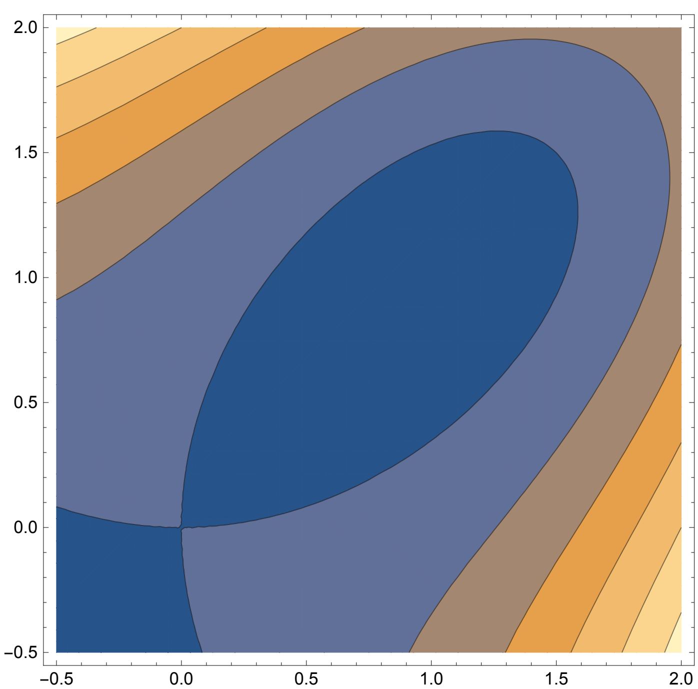
Out[196]= $x^3 - 3\,x\,y + y^3$

Out[201]= $\{3\,x^2 - 3\,y,\ -3\,x + 3\,y^2\}$

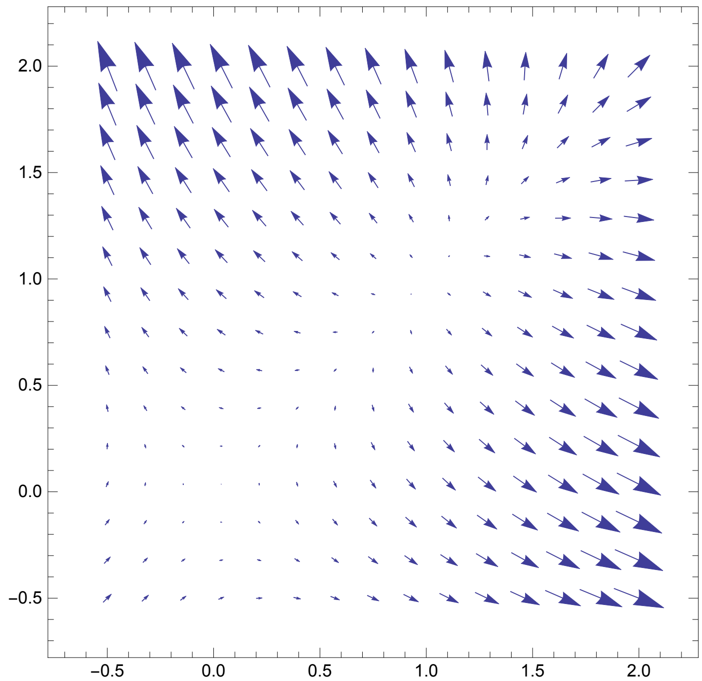
Out[202]=



Out[203]=



Out[204]=



In[207]:=
```
(* at critical points the gradient is 0 or undefined *)
crits = Solve[Gf == {0, 0}]
```

Out[207]= $\{\{x \to 0, y \to 0\}, \{x \to 1, y \to 1\}, \{x \to -(-1)^{1/3}, y \to (-1)^{2/3}\}, \{x \to (-1)^{2/3}, y \to -(-1)^{1/3}\}\}$

In[208]:=
```
(* critical points *)
crits[[1]]
crits[[2]]
```

Out[208]= $\{x \to 0, y \to 0\}$

Out[209]= $\{x \to 1, y \to 1\}$

In[225]:=
```
(* Hessian matrix= derivative of the gradient *)
H = D[Gf, {{x, y}}];
MatrixForm[H]
```

Out[226]//MatrixForm=

$$\begin{pmatrix} 6x & -3 \\ -3 & 6y \end{pmatrix}$$

In[227]:=
```
(* determinant at the Hessian at a critical point *)
HCrit = H /. crits[[2]];
MatrixForm[HCrit]
Det[HCrit]
```

Out[228]//MatrixForm=

$$\begin{pmatrix} 6 & -3 \\ -3 & 6 \end{pmatrix}$$

Out[229]= 27