In[1]:=
```
(* We compute the centroid of the upper
 hemisphere with radius R centered at the origin *)
(* parametrization *)
r = {R Sin[ph] Cos[th], R Sin[ph] Sin[th], R Cos[ph]}
(* partial derivatives *)
rt = D[r, th]
rp = D[r, ph]
(* normal vector *)
n = Simplify[Cross[rt, rp]]
(* length normal vector *)
nn = Simplify[Norm[n], Assumptions -> R > 0 && 0 < th < 2 Pi && 0 < ph < Pi / 2]
(* surface integral of 1 *)
m = Integrate[Integrate[nn, {th, 0, 2 Pi}], {ph, 0, Pi / 2}]
(* surface integral of z *)
mz = Integrate[Integrate[r[[3]] nn, {th, 0, 2 Pi}], {ph, 0, Pi / 2}]
(* centroid *)
zbar = mz / m
```

Out[1]= $\{R \cos[th] \sin[ph], R \sin[ph] \sin[th], R \cos[ph]\}$

Out[2]= $\{-R \sin[ph] \sin[th], R \cos[th] \sin[ph], 0\}$

Out[3]= $\{R \cos[ph] \cos[th], R \cos[ph] \sin[th], -R \sin[ph]\}$

Out[4]= $\left\{-R^2 \cos[th] \sin[ph]^2, -R^2 \sin[ph]^2 \sin[th], -R^2 \cos[ph] \sin[ph]\right\}$

Out[5]= $R^2 \sin[ph]$

Out[6]= $2 \pi R^2$

Out[7]= πR^3

Out[8]= $\frac{R}{2}$