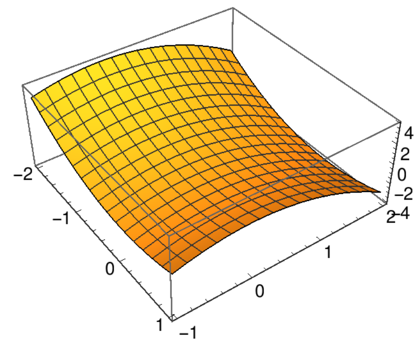
```
(* We find the global extrema of x^2-y^2  on the region [-2,1]x[-1,2] *)
```

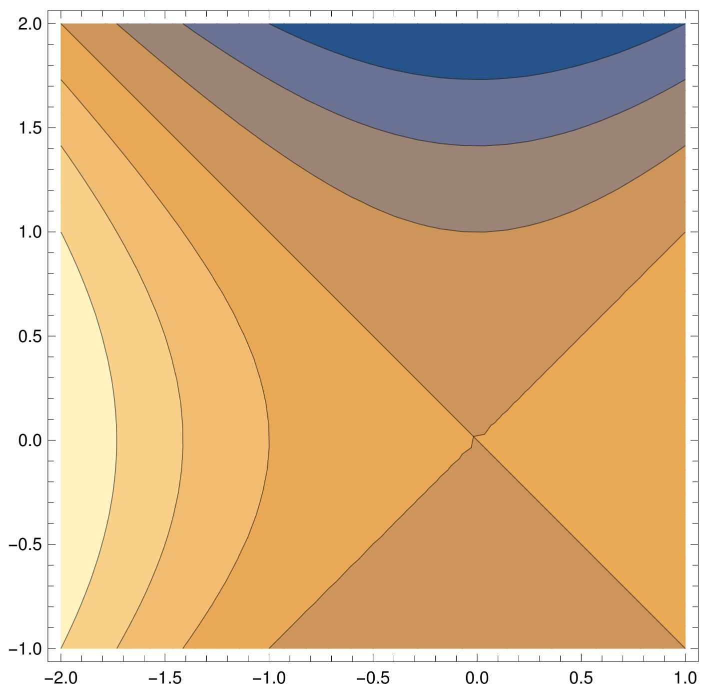
In[1]:=
```
f = x^2 - y^2
Plot3D[f, {x, -2, 1}, {y, -1, 2}]
ContourPlot[f, {x, -2, 1}, {y, -1, 2}]
```

Out[1]= $x^2 - y^2$

Out[2]=



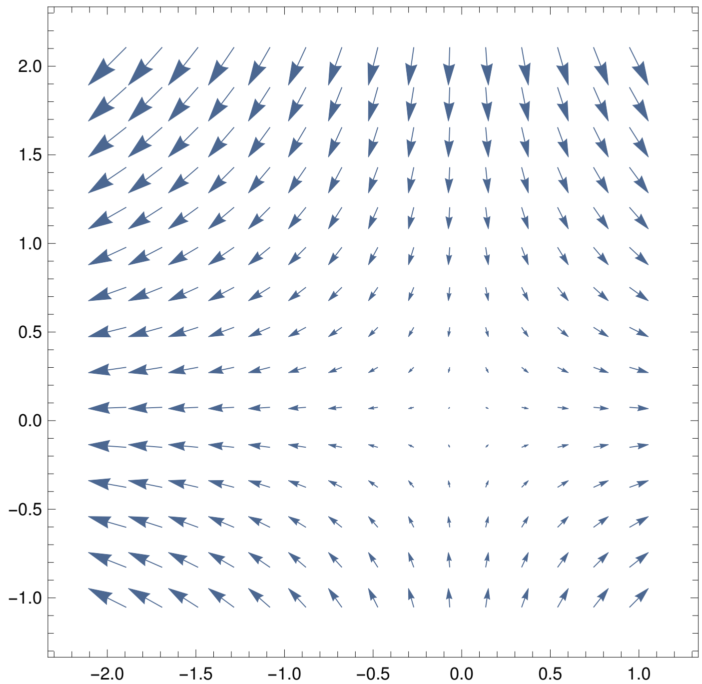
Out[3]=



In[4]:=
```
Gf = Grad[f, {x, y}]
VectorPlot[Gf, {x, -2, 1}, {y, -1, 2}]
```

Out[4]= {2 x, -2 y}

Out[5]=



In[6]:=
```
(* Critical points *)
crits = Solve[Gf == {0, 0}, {x, y}]
```

Out[6]= {{x → 0, y → 0}}

In[7]:=
```
H = D[Gf, {{x, y}}] /. crits[[1]];
MatrixForm[H]
Det[H]
(* The only critical point inside the region is
 a saddle point. The global extrema cannot be there. *)
```

Out[8]//MatrixForm=

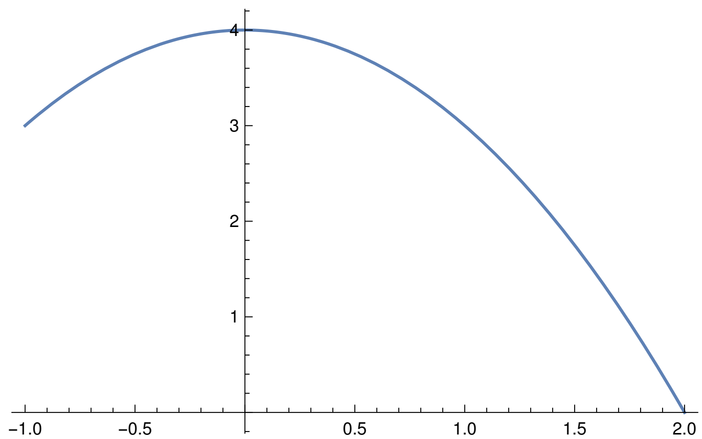
$$\begin{pmatrix} 2 & 0 \\ 0 & -2 \end{pmatrix}$$

Out[9]= -4

In[10]:=
```
(* Boundary *)
west = f /. {x → -2, y → t}
Plot[west, {t, -1, 2}]
wcrits = Solve[D[west, t] == 0, t]
west /. t → -1
west /. wcrits[[1]]
west /. t → 2
```

Out[10]= $4 - t^2$

Out[11]=



Out[12]= {{t → 0}}

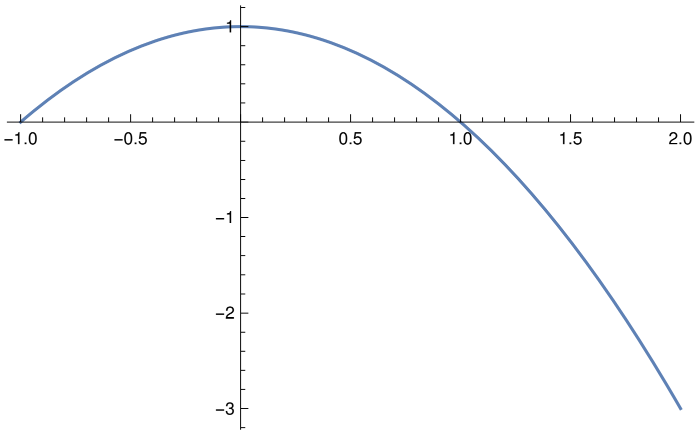
Out[13]= 3

Out[14]= 4

Out[15]= 0

In[16]:=
```
east = f /. {x → 1, y → t}
Plot[east, {t, -1, 2}]
ecrits = Solve[D[east, t] == 0, t]
east /. t → -1
east /. ecrits[[1]]
east /. t → 2
```

Out[16]= $1 - t^2$

Out[17]= 

Out[18]= {{t → 0}}

Out[19]= 0

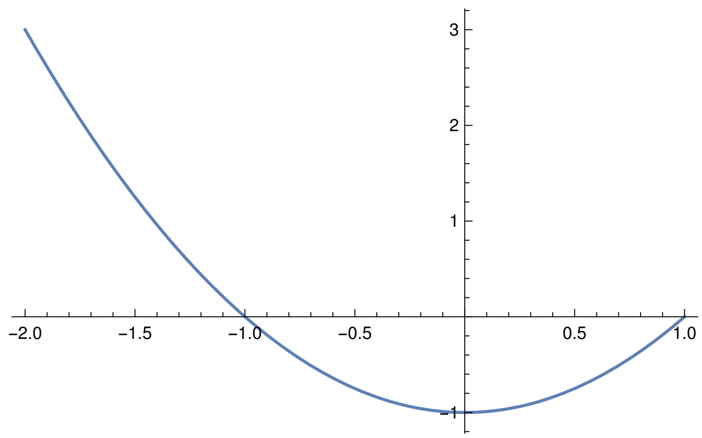
Out[20]= 1

Out[21]= -3

In[22]:=
```
south = f /. {x → t, y → -1}
Plot[south, {t, -2, 1}]
scrits = Solve[D[south, t] == 0, t]
south /. t → -2
south /. scrits[[1]]
south /. t → 1
```

Out[22]= $-1 + t^2$

Out[23]=



Out[24]= {{t → 0}}

Out[25]= 3

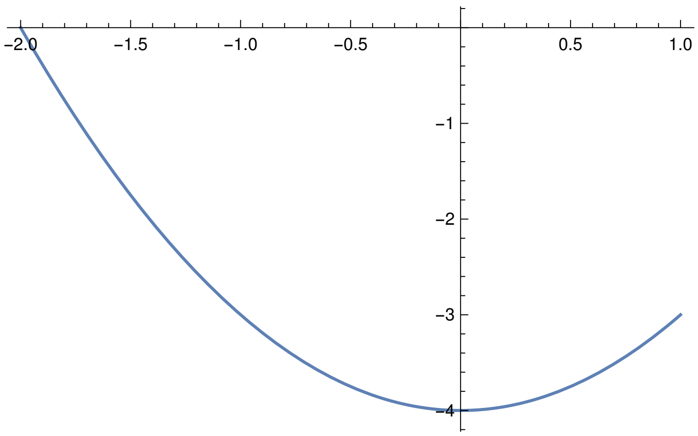
Out[26]= -1

Out[27]= 0

In[28]:=
```
north = f /. {x → t, y → 2}
Plot[north, {t, -2, 1}]
ncrits = Solve[D[north, t] == 0, t]
north /. t → -2
north /. ncrits[[1]]
north /. t → 1
```

Out[28]= $-4 + t^2$

Out[29]=



Out[30]= {{t → 0}}

Out[31]= 0

Out[32]= -4

Out[33]= -3

In[34]:=
```
(* Global maximum of 4 at (-2,0).
   Global minimum of -4 at (0,2) *)
```