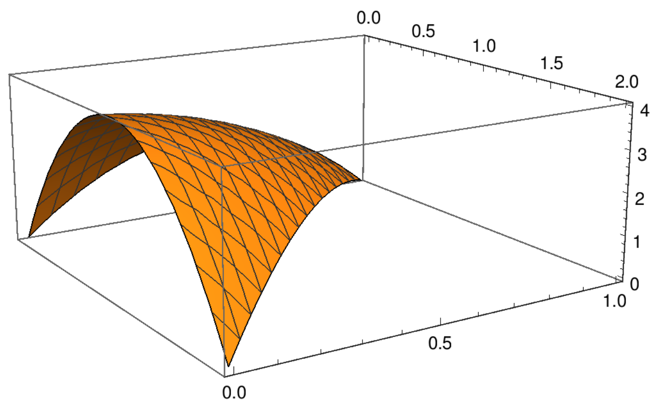
In[1]:=
```
(* We find the maximum surface area of a brick in the first octant
   with diagonal corners at (0,0,0) and on the plane 2x+4y+z=4 *)
```

In[2]:=
```
z = 4 - 2 x - 4 y
S = Expand[2 (x y + x z + y z)] (* surface area *)
Plot3D[S, {x, 0, 2}, {y, 0, 1}, RegionFunction -> Function[{x, y}, 4 - 2 x - 4 y ≥ 0]]
ContourPlot[S, {x, 0, 2}, {y, 0, 1}, Contours → 30,
 RegionFunction -> Function[{x, y}, 4 - 2 x - 4 y ≥ 0]]
```

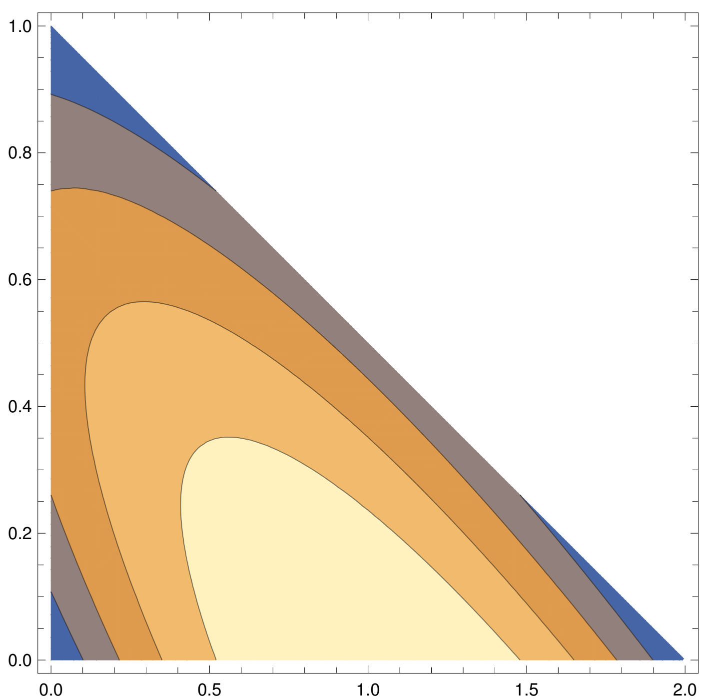
Out[2]= $4 - 2x - 4y$

Out[3]= $8x - 4x^2 + 8y - 10xy - 8y^2$

Out[4]=



Out[5]=



In[6]:=
```
G = Grad[S, {x, y}] (* Find the critical points *)
crit = Solve[G == {0, 0}, {x, y}]  (* No critical point in the allowed region *)
S /. crit[[1]]
```

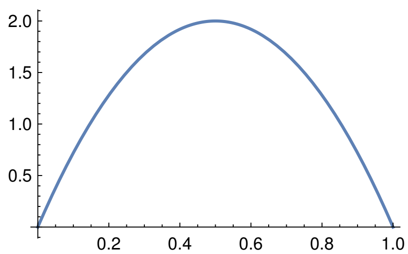
Out[6]= $\{8 - 8x - 10y, 8 - 10x - 16y\}$

Out[7]= $\left\{\left\{x \to \frac{12}{7}, y \to -\frac{4}{7}\right\}\right\}$

Out[8]= $\frac{32}{7}$

In[9]:=
```
(* Need to consider the three boundary curves *)
f = S /. {x → 0, y → t}
Plot[f, {t, 0, 1}]
cand1 = Solve[D[f, t] == 0]
f /. cand1[[1]]
```

Out[9]= $8t - 8t^2$

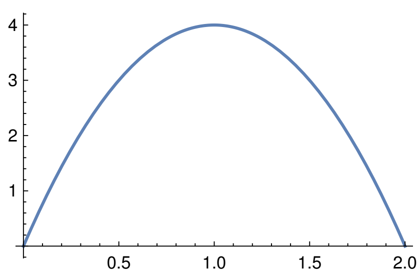
Out[10]=


Out[11]= $\left\{\left\{t \to \frac{1}{2}\right\}\right\}$

Out[12]= 2

In[13]:=
```
g = S /. {x → t, y → 0}
Plot[g, {t, 0, 2}]
cand2 = Solve[D[g, t] == 0]
g /. cand2[[1]]
```

Out[13]= $8t - 4t^2$

Out[14]=


Out[15]= $\{\{t \to 1\}\}$

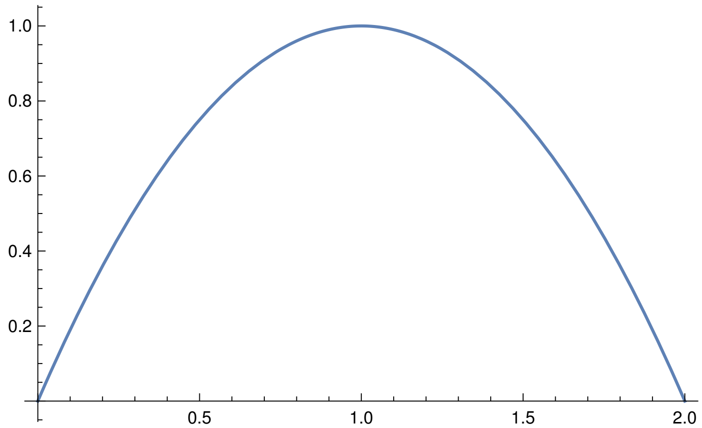
Out[16]= 4

In[17]:=
```
yy = Solve[z == 0, y]  (* express y in terms of x assuming z=0 *)
h = Expand[S /. yy[[1]] /. {x → t}]
Plot[h, {t, 0, 2}]
cand3 = Solve[D[g, t] == 0]
h /. cand3[[1]]
```

Out[17]= $\left\{\left\{y \to \frac{2-x}{2}\right\}\right\}$

Out[18]= $2\,t - t^2$

Out[19]=



Out[20]= $\{\{t \to 1\}\}$

Out[21]= 1

In[22]:=
```
(* The global maximum is 4 when x=1, y=0, z=2 *)
S /. {x → 1, y → 0}
z /. {x → 1, y → 0}
```

Out[22]= 4

Out[23]= 2