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In[109]:= (* 3D vector field *)
F = {2 x + 3 y + 4 z, x^2 + y^2 + z^2, x^3 + y^3 + z^3}
(* sphere around (x,y,z) with radius r *)
q = {x + r Sin[ph] Cos[th], y + r Sin[ph] Sin[th], z + r Cos[ph]};
volume = 4 Pi r^3 / 3;

Out[109]= {2 x + 3 y + 4 z, x^2 + y^2 + z^2, x^3 + y^3 + z^3}

In[124]:= (* flux density *)
qth = D[q, th];
qph = D[q, ph];
n = Simplify[Cross[qph, qth]]
Foq = Simplify[F /. {x -> q[[1]], y -> q[[2]], z -> q[[3]]}];
flux = Integrate[Integrate[Dot[Foq, n], {th, 0, 2 Pi}], {ph, 0, Pi}]
Limit[flux / volume, r -> 0]

Out[126]= {r^2 Cos[th] Sin[ph]^2, r^2 Sin[ph]^2 Sin[th], r^2 Cos[ph] Sin[ph]}

Out[128]=  $\frac{4}{15} \pi r^3 (3 r^2 + 5 (2 + 2 y + 3 z^2))$ 

Out[129]=  $2 + 2 y + 3 z^2$ 

In[93]:= Div[F, {x, y, z}]

Out[93]=  $2 + 2 y + 3 z^2$ 

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