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In[1]:= (* We compute the surface area of the upper
hemisphere with radius R centered at the origin *)
q = {r Cos[th], r Sin[th], Sqrt[R^2 - r^2]};
qr = D[q, r];
qth = D[q, th];
n = Simplify[Cross[qr, qth]]
nn = Simplify[Norm[n], Assumptions -> R ≥ 0 && R ≥ r && r ≥ 0 && th ≥ 0 ]
Assuming[R > 0,
Integrate[Integrate[nn, {th, 0, 2 Pi}], {r, 0, R}]
]

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$$\text{Out[4]} = \left\{ \frac{r^2 \cos[th]}{\sqrt{-r^2 + R^2}}, \frac{r^2 \sin[th]}{\sqrt{-r^2 + R^2}}, r \right\}$$

$$\text{Out[5]} = \frac{r R}{\sqrt{-r^2 + R^2}}$$

$$\text{Out[6]} = 2 \pi R^2$$