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In[1]:= (* We compute the surface area of the upper
hemisphere with radius R centered at the origin *)
q = {u, v, Sqrt[R^2 - u^2 - v^2]};
qu = D[q, u];
qv = D[q, v];
n = Simplify[Cross[qu, qv]]
nn = Simplify[Norm[n], Assumptions -> R ≥ 0 && R^2 ≥ u^2 + v^2]
T = {u → r Cos[th], v → r Sin[th]};
a = Simplify[r nn /. T, Assumptions -> R ≥ 0 && r ≥ 0 && R ≥ r]
Assuming[R > 0,
  Integrate[Integrate[a, {th, 0, 2 Pi}], {r, 0, R}]
]

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$$\text{Out[4]} = \left\{ \frac{u}{\sqrt{R^2 - u^2 - v^2}}, \frac{v}{\sqrt{R^2 - u^2 - v^2}}, 1 \right\}$$

$$\text{Out[5]} = \frac{R}{\sqrt{R^2 - u^2 - v^2}}$$

$$\text{Out[7]} = \frac{r R}{\sqrt{-r^2 + R^2}}$$

$$\text{Out[8]} = 2 \pi R^2$$