MAT 136
Calculus I Lecture Notes

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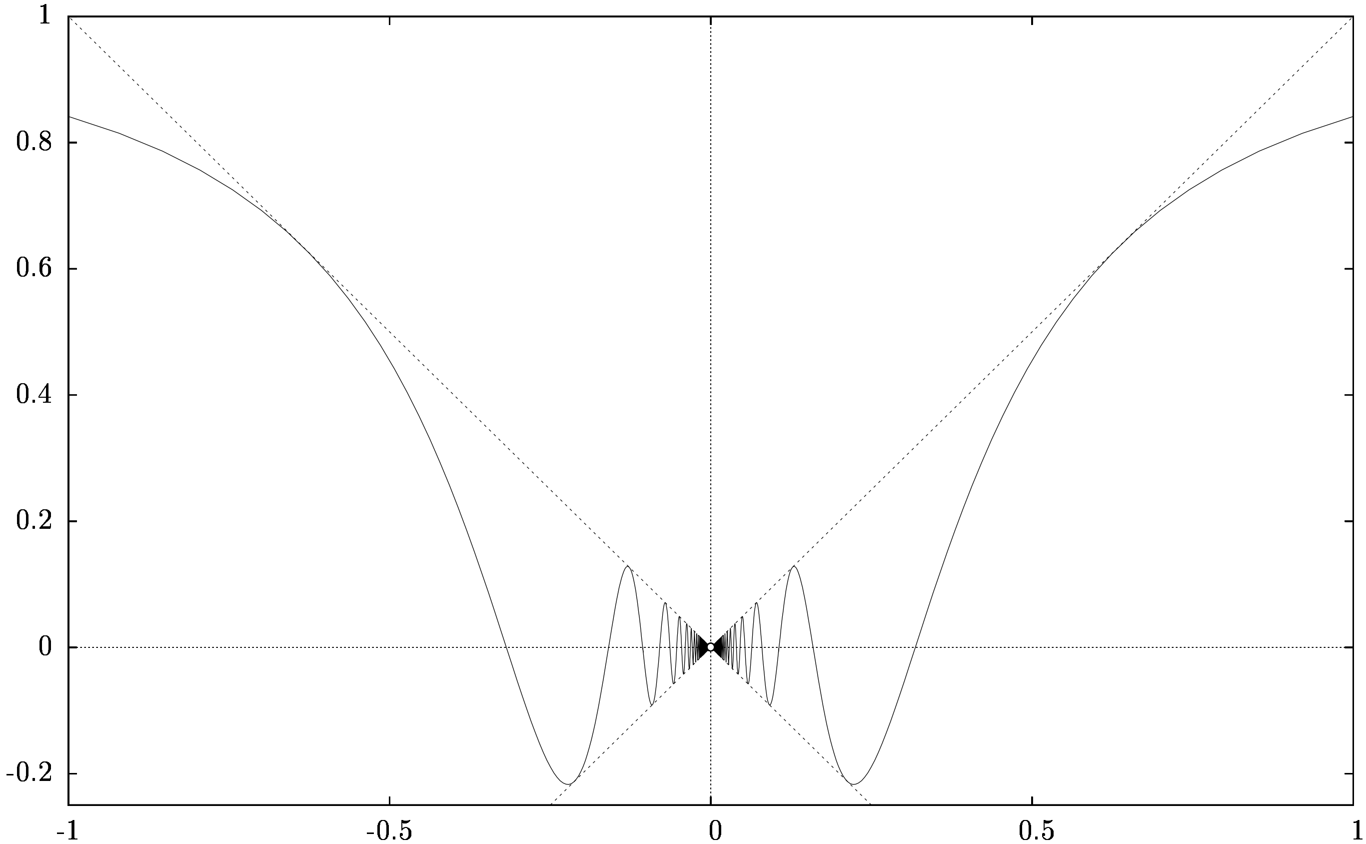

$$x \mapsto x \sin\left(\frac{1}{x}\right)$$

 http://jan.ucc.nau.edu/ns46/CLN

Fill in the table with the derivatives. Extend the table with your favorite derivatives.

$f(x)$	$f'(x)$
c	
$\frac{1}{x}$	
$\ln(x)$	
$\sin(x)$	
$\sinh(x)$	
$\tan(x)$	

$f(x)$	$f'(x)$
ax	
x^n	
$\log_b(x)$	
$\arcsin(x)$	
$\operatorname{arcsinh}(x)$	
$\arctan(x)$	

$f(x)$	$f'(x)$
$\|x\|$	
e^x	
$\ln(\|x\|)$	
$\cos(x)$	
$\cosh(x)$	
$\tanh(x)$	

$f(x)$	$f'(x)$
$\sqrt{x}$	
a^x	
x^x	
$\arccos(x)$	
$\operatorname{arccosh}(x)$	
$\operatorname{arctanh}(x)$	

Fill in the table with the derivative rules.

$(cf)' =$	
$\left(\frac{1}{f}\right)' =$	

$(f+g)' =$	
$(fg)' =$	

$(f-g)' =$	
$\left(\frac{f}{g}\right)' =$	

$\frac{d}{dx} f(g(x)) =$	
$\frac{d}{dx} f^{-1}(x) =$	

Fill in the table with the integral rules.

$\int x^n \, dx =$	
$\int cf =$	
$\int_a^b f' =$	
$\int_a^b f + \int_b^c f$	

$\int \frac{1}{x} \, dx =$	
$\int (f+g)$	
$\int f'(ax+b) \, dx =$	
$\int f'(g(x))g'(x) \, dx =$	

$\int \frac{f'(x)}{f(x)} \, dx =$	
$\int f'g =$	
$\frac{d}{dx} \int_a^x f(t) \, dt =$	
$\frac{d}{dx} \int_a^{g(x)} f(t) \, dt =$	

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Introduction

These lecture notes are intended to be a concise treatment of first semester calculus. As a result, this book is light weight and easy to carry to school but the presented material is somewhat more dense than usual. It is not a complete self study guide. It relies on the explanations provided by a teacher. The number of examples and exercises are kept small enough to be easily remembered but extensive enough to help with understanding the concepts. For full mastery of the material the book should be supplemented by WeBWorK exercise sets.

In every mathematics class, the goal is to understand the material as deeply as possible. If you find yourself spending your time on too much memorization, then you are doing something wrong. For example, it is not a good idea to write the derivative formulas on flash cards and memorize them. It is much more helpful to find the derivatives of a few functions every morning when you wake up and every night before you go to sleep and look up the formulas you do not yet remember. Spending time on solving problems, understanding the reasons behind definitions and theorems, analyzing a few important examples should result in automatic recall of the required lexical knowledge without memorization. Many mathematicians have terrible memory. Perhaps this is why they became mathematicians and not botanists. The approach of mathematics is to find a small amount of essential knowledge that allows us to handle the most problems. We do not learn the answer for every possible question. That would be impossible. We learn how to answer questions we have not heard before.

Reading a mathematics book is different from reading a novel. It is done much more slowly and carefully. You should always ask questions while you read. Why is this true? Why is this defined this way and not that way? It is most satisfying to answer these questions yourself, but do not be afraid to ask your teacher if you do not understand something. The most important role of your teacher is to answer your questions. A mathematics book is usually not read from front to back. There are things in it you already know. There are things you do not understand immediately. You may have to come back to it several times. Perhaps an example later in the book makes it clear why something was needed earlier.

Solving problems is essential for learning mathematics. Do not be surprised if solving a problem in the book takes a lot of time or takes several attempts. There is nothing wrong about that. What is important is spending time on thinking about problems. Learning what does not work is just as important as learning what does. A solution to a problem presented to you by someone might seem very easy but you might forget it in a couple of days just as easily. Solutions you found yourself is going to stay with you for a long time because you earned it with the hard work you invested in solving it.

Some of the presented proofs are completely rigorous, some are very far from an actual proof, but most should be considered *proofoids* that have the main idea of the proof and in fact can be made into a proof with some extra effort. The proofs are just as important as the theorems themselves.

Problems are a bit harder than the routine exercises. They are more interesting as well, so it is worth the effort to solve them.