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My first result with Alfonso Castro and Jorge Cossio, from my dissertation, proved the existence of a solution $u : \Omega \to \mathbb{R}$ to the superlinear elliptic boundary value problem

$$\Delta u + f(u) = 0, \tag{1}$$

with zero Dirichlet boundary conditions and the nonlinearity $f$ satisfying certain standard superlinear conditions. This solution to the partial differential equation PDE (1) minimizes the action functional over the set of sign changing solutions, and changes sign exactly once. Subsequent papers extended or applied this result for other analysis. Several of my early papers used steepest descent in Sobolev space with projections onto subsets of manifolds in function space to effect the modified mountain pass algorithm for finding minimal energy sign changing solutions numerically. With Jim Swift, the gradient Newton-Galerkin algorithm (GNGA) was developed to find numerically, in principal and within reason, all solutions to a wide class of nonlinear elliptic boundary value problems including (1).

More recently, we have applied modifed versions of the GNGA to easily find new solutions near bifurcation points and follow the corresponding new, bifurcating branches. In fact, we seek solutions to partial *difference* equations (PdE) of the form

$$-Lu + f_s(u) = 0, \tag{2}$$

where $L$ is the matrix for the graph Laplacian on a graph $G$ and vectors $u \in \mathbb{R}^n$ are identified with functions that map $G \to \mathbb{R}$. We have applied this methodology to large graphs that approximate PDE on regions with fractal boundaries, and smaller graphs that have large predescribed symmetry groups. Many of the possible types of bifurcations are successfuly observed.

Current research directions include extensions and applications of our major algorithm and accompanying packages of mathematics utilities. We are applying our techniques to very large graphs, approximating solutions to PDE. When the graphs are pre-fractal, the PDE is defined on a fractal domain. Student projects have applied the technique to studying several types of systems of PdE. Efforts to prove the existence of additional sign changing solutions continue, but are difficult.

Much of our current and past research has been done in collaboration with students, some funded by an NSF or other type grant. To date, three of seven M.S. students have published their thesis results. Many REU, independent study, and undergraduate research students have contributed to the effort as well; for example in coding for parallel networks and doing experiments in Mathematica, MATLAB, and C++ for neural networks, ODE, PDE and PdE.