MAT 226 Instructor: M. Keindl **Test 3. Practice worksheet**

These are just a couple of extra problems for you to work on. When preparing for the test, look over your quizzes/HW and also the examples, definitions, theorems and proofs from class. Test 3 covers graph theory.

1. Find a directed graph that is connected but not strongly connected.
2. The wheel W_n is an n-cycle with an extra vertex that is connected to every other vertex by an edge.
a. How many vertices and edges does W_n have?
b. For what n is W_n bipartite?
c. For what n is W_n a tree?
d. For what n is W_n planar?
e. Find the adjacency matrix of W_4
f. How many circuits of length 3 are there in W_3?
g. For all n that makes W_n planar, determine the number of regions, W_n divides the plane into.
h. For what n does W_n have an Euler circuit?
i. For what n does W_n have a Hamiltonian cycle?
j. Can we erase a vertex of W_{n+1} to make it isomorphic to W_n?
3. How many edges does a forest with m components and n vertices have?

Detailed list of things to know for the test:
- simple graph, pseudograph, incidency, adjacency, degree of a vertex, loop, subgraph, complete graph, bipartite graph, degree sequence, graph isomorphism

- walk, path, trail, circuit, cycle, Euler-circuit, connected graph, Euler pseudograph (Theorem about it), Hamilton cycle, Hamilton graph, Dirac's theorem

- adjacency matrix A, (i,j)th entry of A^n, finding a permutation matrix for isomorphic graphs such that $A = PBP^T$, weighted graphs, traveling salesman's problem, what is Dijkstra's algorithm used for

- directed graphs, arcs, indegree, outdegree, strongly connected graph, Euler digraph (theorem about it), adjacency matrix of digraphs, tournament, transitive tournament, Rédei's theorem, 3 equivalent properties for a tournament

- trees (minimally connected, maximally cycle free), rooted tree, forest, representing a tree with a sequence of $n-1$ numbers, number of labeled trees with n vertices, Kruskal's algorithm

- planar graphs, Euler's formula for planar graphs, inequality between edges and vertices, homeomorphic graphs, Kuratowski's theorem

- $\sum_{A \epsilon V} degA = 2E$

- number of odd vertices is even

- $\mathcal{G}_1 \cong \mathcal{G}_2 \Longrightarrow$ same number of vertices, same number of edges, same degree sequence
(also modified properties for directed graphs)

- A pseudograph is Eulerian $\Longleftrightarrow$ it is connected and every vertex is even
(also modified properties for digraphs)

- $\sum$ indegree $= \sum$ outdegree $=$ number of arcs

- all other theorems mentioned in above list