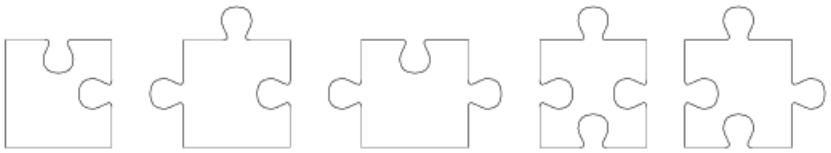
299 SPRING 2009 **HOMEWORK 4**

Turn in your solution to at least two of the problems. **Explain your solution in full sentences. Give detailed reasonings.** Include diagrams and figures if appropriate.

Problem 1. A rectangular puzzle that says "850 pieces" actually consists of 851 pieces. Each piece is identical to one of the 5 samples shown in the diagram. How many pieces of type (E) are there in the puzzle?



Problem 2. The inhabitants of a certain planet use not four but five basic arithmetic operations. The operations of addition, multiplication, subtraction and division are the same as ours, but they also have a special operation denoted by the sign @. We do not know exactly how this operation works, but we have found out that the following properties are valid for all x and y:

(a) $x@0 = x$
(b) $x@y = y@x$
(c) $(x+1)@y = (x@y) + y + 1$

What is the value of 12@5 on this planet?

Problem 3. Add all positive integers for which the quotient and the remainder are equal if the number is divided by 2009. What is the result?